A spatio-temporal study of extreme rainfall in Guanajuato State, Mexico.

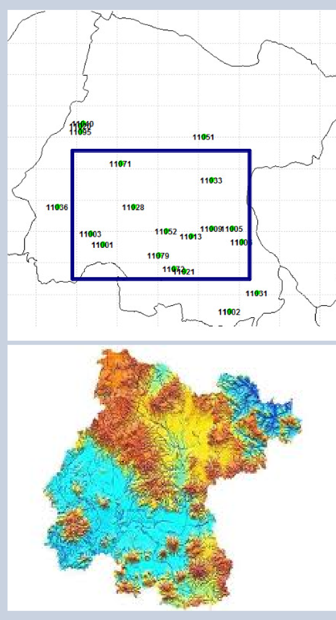
Moreno Leonardo - Ortega Joaquín
mrleo@iesta.edu.uy -jortega@cimat.mx

UNIVERSIDAD DE LA REPUBLICA
CIMAT

Abstract

The aim is to establish a space-time model for the maximum values of daily rainfall in the state of Guanajuato, using measurements recorded at meteorological stations in the region. While the natural way to extend the theory from one-dimensional extremes to the spatial case are the max-stable processes, inference on that class of processes is not very flexible. This work shows the close relation between the finite-dimensional distributions of max-stable processes and extreme copulas. Predictions under both models are compared, using different max-stable processes and r-vines, where the latter provide a possible solution to the problem of non-stationarity.

Data set



- There is a total of 160 measuring stations in the state, some of which have data as far back as 1902. We considered data up to 2012. Stations having more than 10% missing data were discarded. In order to deal with the problem of missing data, two shorter periods were considered: dry (November-March) and wet (June-August). The orography directly influences the rainfall distribution. A simulation study was carried out for the region bounded by the blue rectangle in the map.

Max-stable process

- Let $Z \in C(K)$ a stochastic process with non-degenerate marginal. Z is a max-stable process if given n independent copies $Z_1, \ldots, Z_n$ there are constants $a_n > 0$ and $b_n \in \mathbb{R}$ such that,

$$Z \stackrel{d}{=} \text{máx}_{i=1,\ldots,n} \frac{Z_i - b_n}{a_n}, \quad \forall n \in \mathbb{N}.$$

- Let $X_1, X_2, \ldots$ be a sequence of random fields such that,

$$\left\{ \text{máx}_{i=1,\ldots,n} \frac{X_i(x) - b_n(x)}{a_n(x)} \right\}_{x \in K} \xrightarrow{d} \{Y(x)\}_{x \in K},$$

Where Y is non-degenerate, with $a_n, b_n \in C(K)$, $K \subset \mathbb{R}^d$ compact and $a_n > 0$.

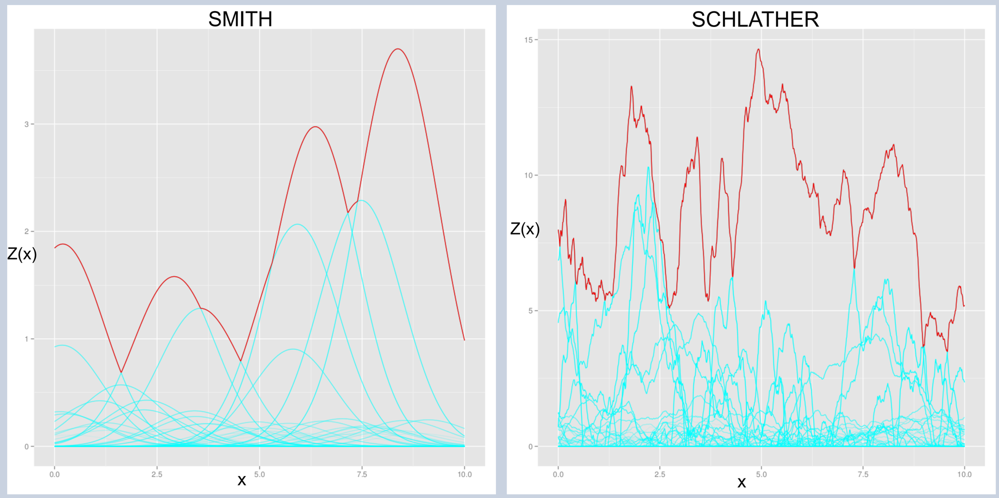
- Then the class of limit processes coincides with the class of max-stable processes.

Characterization.

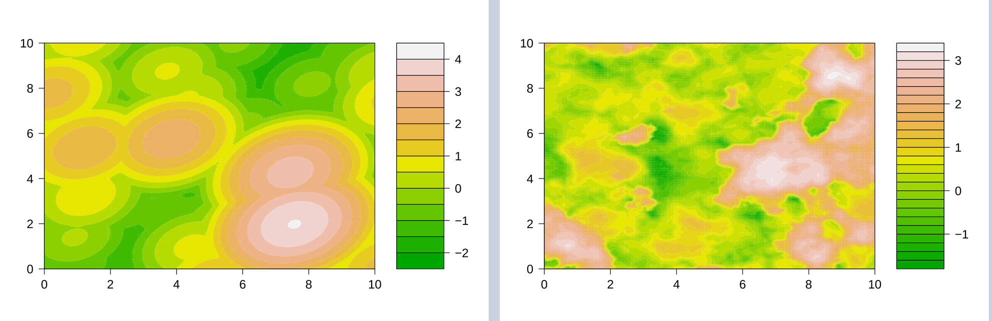
- **[Schlather, 2002]** Let $\{\psi_i : i \in \mathbb{N}\}$ be a Poisson point process on $(0, \infty)$ with intensity $d\Lambda(\psi) = \psi^{-2} d\psi$. and let Y be a independent, non-negative stochastic process Y with continuos trajectories, such that $E(Y(x)) = 1$, $\forall x \in \mathbb{R}^d$. The process $Z(x) = \text{máx}_{i \geq 1} \psi_i Y_i(x)$, $x \in X$ where $Y_1, Y_2, \ldots$ are independent copies of Y, is a max-stable process.

Examples

- Smith and Schlather processes $\mathbb{R}$.



- Level curves for Smith and Schlather processes in $\mathbb{R}^2$.



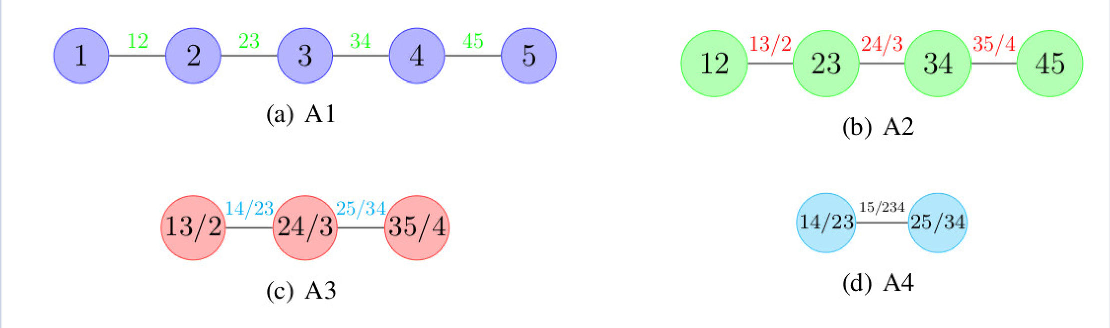
Extreme value copulas

- A copula can be thought of as the distribution function of a vector with $U(0,1)$ marginal.
- C_* is a extreme value copula if there is another copula C satisfying

$$C\left(u_1^{1/n}, u_2^{1/n}, \ldots, u_d^{1/n}\right)^n \to C_*(u_1, u_2, \ldots, u_d),$$

when $n \to +\infty$ y $\forall (u_1, u_2, \ldots, u_d) \in [0,1]^d$.

Example: D-vines in dimension 5



$f_{12345} = f_1 f_2 f_3 f_4 f_5 c_{12} c_{23} c_{34} c_{45} c_{13/2} c_{24/3} c_{35/4} c_{14/23} c_{25/34} c_{15/234}$

Regular Vines

A set of trees $\mho = (T_1, \ldots, T_{d-1})$ is a vine, [Kurowicka and Joe, 2011], if and only if,

- T_1 is a tree with nodes $N_1 = \{1, \ldots, d\}$ and edges E_1.
- For $i = 2, \ldots, m$, T_i is a tree with nodes $N_i = E_{i-1}$ and a set of edges E_i.
- Two nodes in the $j+1$ tree are joined by an edge if the edges share a node in the $j - th$ tree.

A regular vine is defined as,

- A Vine structure.
- Every edge corresponds to the density of a pairs copula. In this work extreme copulas are considered.
- The density is defined as the pairwise product copulas identified with the edges of the regular vine and the marginal densities.

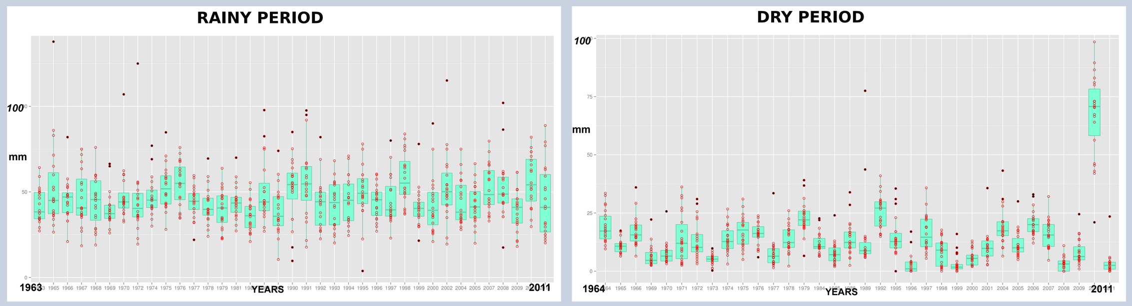
Regular is highly flexible in asymmetries, positive/negative dependence and tail dependence.

Methodology

The annual maxima for each station and each period were obtained and the GEV parameters were estimated using maximum likelihood for the rainy period and using a linear model with latitude, longitude and height as independent variables for the dry period, [Padoan et al., 2010]. These distributions are transformed to a unit FrÃ©chet distribution. The parameters for the max-stable processes were estimated using pairwise maximum likelihood. The R-vines structure was obtained using a sequential procedure introduced by [DíſMann et al., 2013] and the pairwise dependence was modeled using the t-extremal and Gumbel copulas.

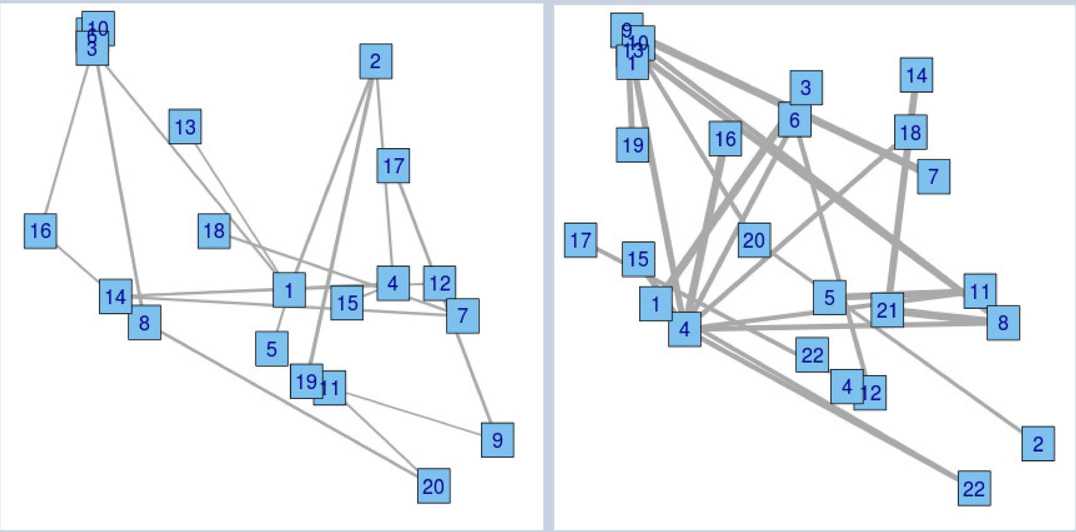
Results.

Boxplots for maximal rainfall for each station during the rainy and dry periods, respectively.



Regular Vines

- First trees for the rainy and dry periods, respectively.



- Predictions for the R-vines model.

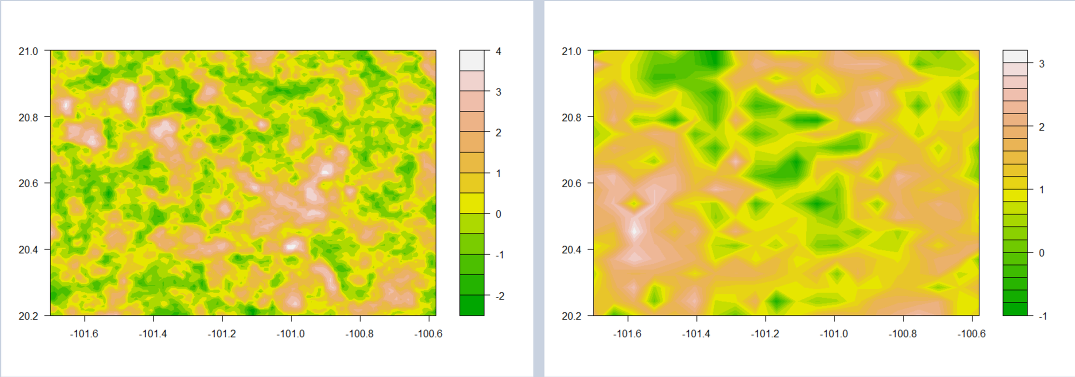
period (years)	*June – August*		*November – March*	
	P^*(exceed 100 mm)	P^*(exceed 150 mm)	P^*(exceed 100 mm)	P^*(exceed 150 mm)
10	0.625	0.063	0.205	0.086
20	0.87	0.121	0.384	0.155
30	0.948	0.161	0.479	0.219
40	0.98	0.194	0.62	0.297
50	0.994	0.268	0.683	0.31

Results: Max-stable processes

- Selecting the max stable process using the criterion of TIC in the rainy period. In the model Schlather's covariance various functions are used.

Models	Estimated Parameters (July-August)				TIC
	nugget	range	smooth	other	
Schlather (Whitmat)	0.152557265	0.004482092	15.365008469		61250.89
Schlather(PowExp)	0.15864611	0.03495404	1.99583921		61250.82
tExtremal	0.003024729	0.040675800	1.219398640	g.l=5	60632.15
Geo. Gaussian	0.001076448	0.044292382	1.230667966	σ^2= 7.845857248	60635.03
Brown Resnick		0.001147649	0.331258917		60645.38

- Simulations using the t-extremal process in both dry and wet periods respectively. Observed different patterns of rainfall.



- Predictions by processes max-stables.

period (years)	*June – August*		*November – March*	
	P^*(exceed 100 mm)	P^*(exceed 150 mm)	P^*(exceed 100 mm)	P^*(exceed 150 mm)
10	0.607	0.03	0.146	0.043
20	0.84	0.11	0.306	0.093
30	0.92	0.13	0.407	0.125
40	0.968	0.195	0.492	0.132
50	0.996	0.234	0.592	0.178

Conclusions

- In the rainy season the process is not spatial stationary. There is a high independence between stations that are nearby. In the dry period the spatial dependence is increased. Predictions using R-vines and using max-stable are similar. The R-vines methodology provides a possible solution to non spatial stationarity problem.
- From November to March the distribution has heavier tails. It is important to study these months since there are relatively higher probabilities of very extreme values.
- T-extremal models exhibit a high flexibility, which makes them very useful in modeling. They are the models that best fit in most cases. These results are consistent since the finite dimensional distributions of t-extremal processes are the t-extremal copulas.
- Global forecasts show that extreme precipitation causing severe flooding in the Guanajuato State are expected in relatively short periods of time.

References

[DíſMann et al., 2013] DíſMann, J., Brechmann, E. C., Czado, C., and Kurowicka, D. (2013). Selecting and estimating regular vine copulae and application to financial returns. *Computational Statistics & Data Analysis*, 59:52–69.

[Kurowicka and Joe, 2011] Kurowicka, D. and Joe, H. (2011). *Dependence Modeling. Vine Copula Handbook*. World Scientific.

[Padoan et al., 2010] Padoan, S. A., Ribatet, M., and Sisson, S. A. (2010). Likelihood-based inference for max-stable processes. *Journal of the American Statistical Association*, 105(489):263–277.

[Schlather, 2002] Schlather, M. (2002). Models for stationary max-stable random fields. *Extremes*, 5(1):33–44.

Acknowledgements

- A la Comisión Nacional del Agua de México, CONAGUA, por permitirnos acceder a la base de datos.
- A la Scapa, al Cimat y al Iesta por el apoyo a la realización de la tesis de maestría de Leonardo Moreno.
- A Enrique Cabaña y Ernesto Mordecki por auspiciar de tutores de tesis y académico respectivamente.